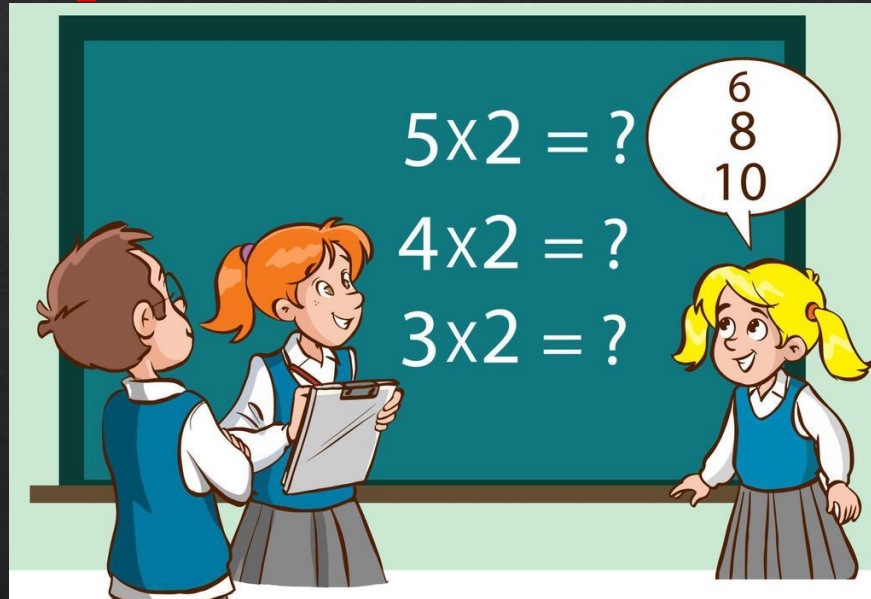




Coral Coast 2026

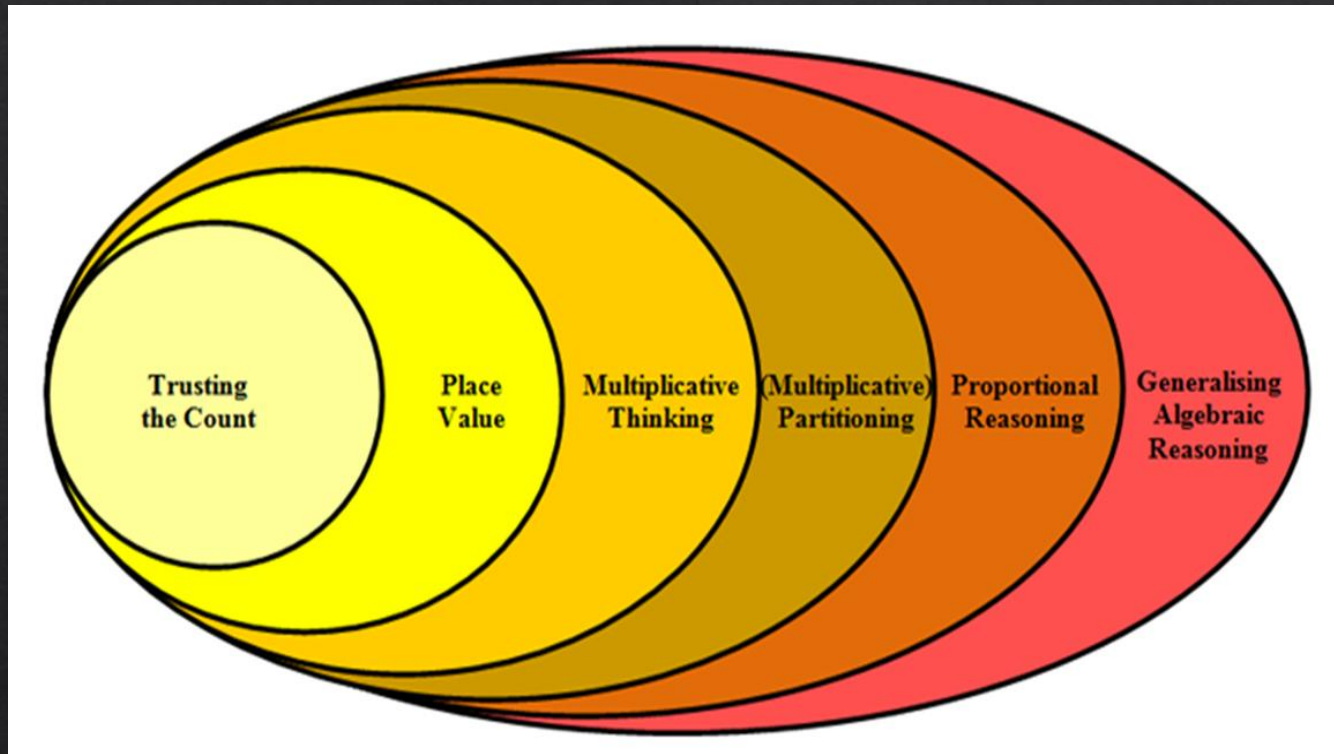
Developing Multiplicative Thinkers



Dr. Chris Hurst
MAWA

hurstc1949@gmail.com

Big Ideas of Number



Hurst & Hurrell, 2014, adapted from Siemon, Bleckley & Neal, 2012

Defining Multiplicative Thinking

- Work flexibly with a wide range of numbers including very large and small whole numbers, decimals, fractions, ratio and percentage.

Defining Multiplicative Thinking

- Work conceptually with the relative magnitude of whole and decimal numbers in a range of representations, demonstrating an understanding of the notion of 'times as many'

Defining Multiplicative Thinking

- Demonstrate a conceptual understanding of the multiplicative situation, the relationship between multiplication and division, numbers of equal groups, factors and multiples, and the various properties of multiplication.

Defining Multiplicative Thinking

- Articulate a conceptual understanding of a range of multiplicative ideas in a connected way with explicit language and terminology.

Hurst & Hurrell (2017), adapted from Siemon, Bleckley,
& Neal (2012)

M/T is fundamental to the development of key concepts

Most of the content descriptors in the SCSA Western Australia Curriculum are underpinned by aspects of multiplicative thinking.



A hard truth . . .

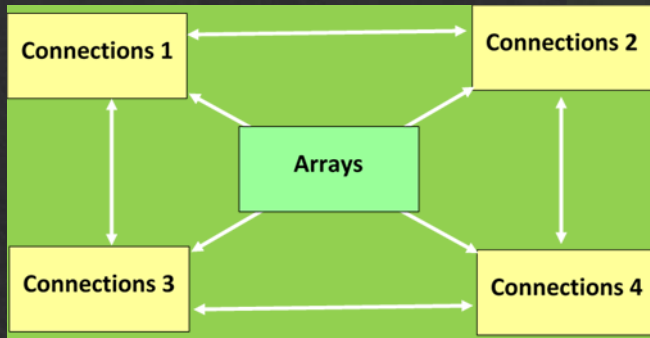
Students who are not well established with multiplicative thinking do not have the foundational knowledge and skills needed to participate effectively in further school mathematics, or to access some post-compulsory training opportunities.

Siemon, Breed, Dole, Izard, & Virgona, 2006

Importantly...

- This is a K to 9 (and beyond) issue.
- The foundations for M/T are of critical importance and need to be built upon as children move through primary school and beyond.

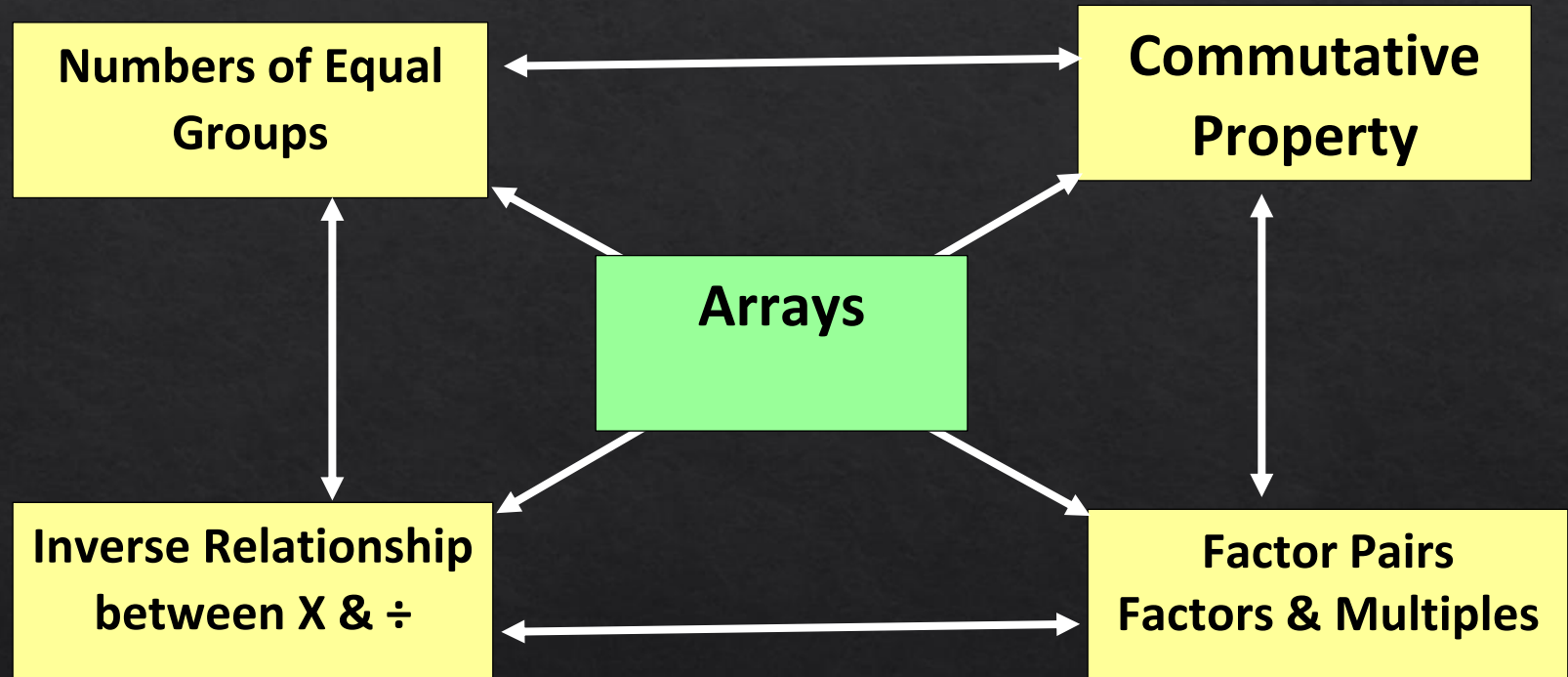




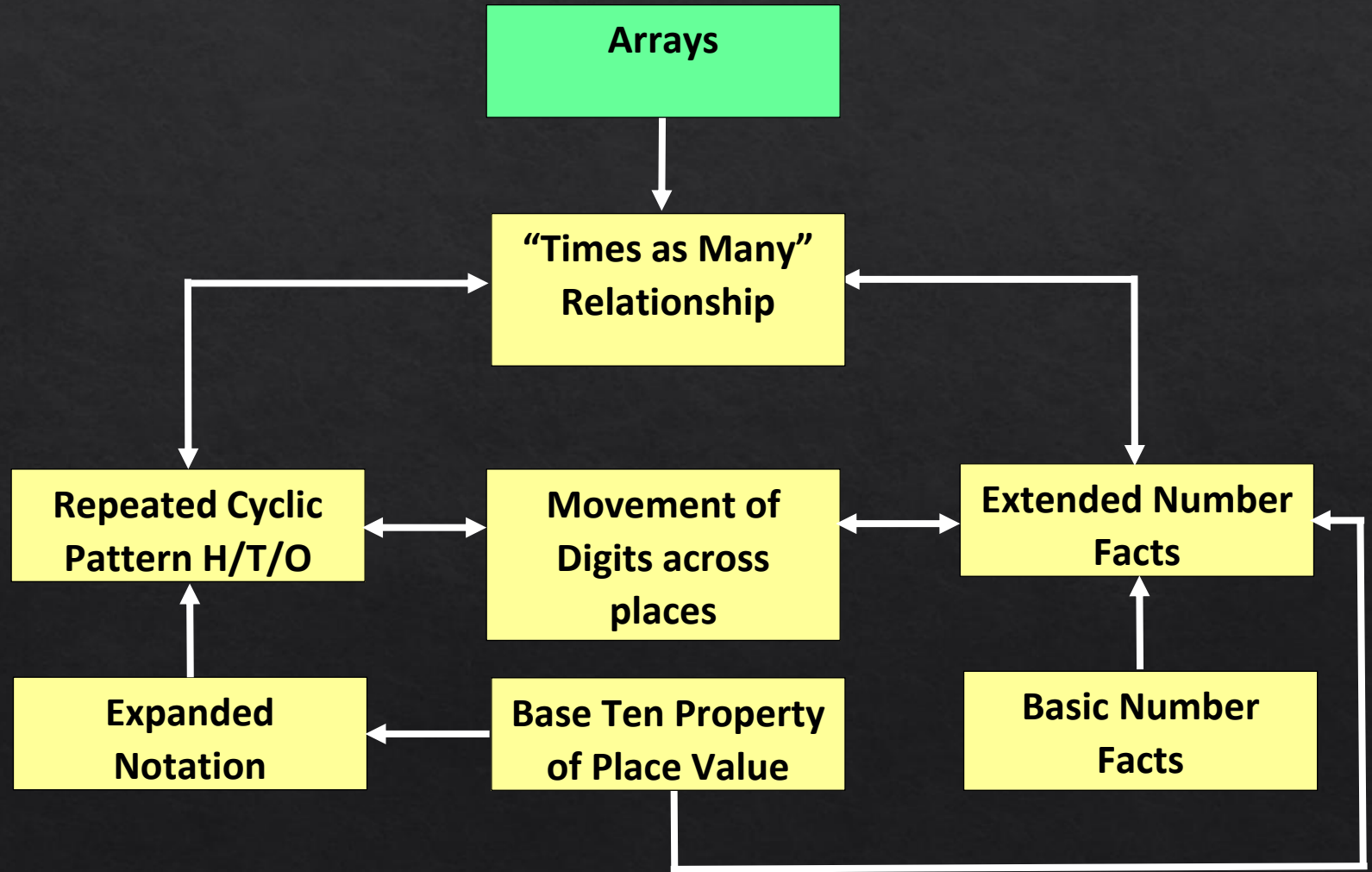
Connections

- **Multiplicative thinking is characterised by four sets of conceptual connections, all of which overlap to varying degrees with at least one other set of connections.**
- **Each set of connections is based on the multiplicative array which can be used to conceptually explain elements of the connection sets and the links between them.**

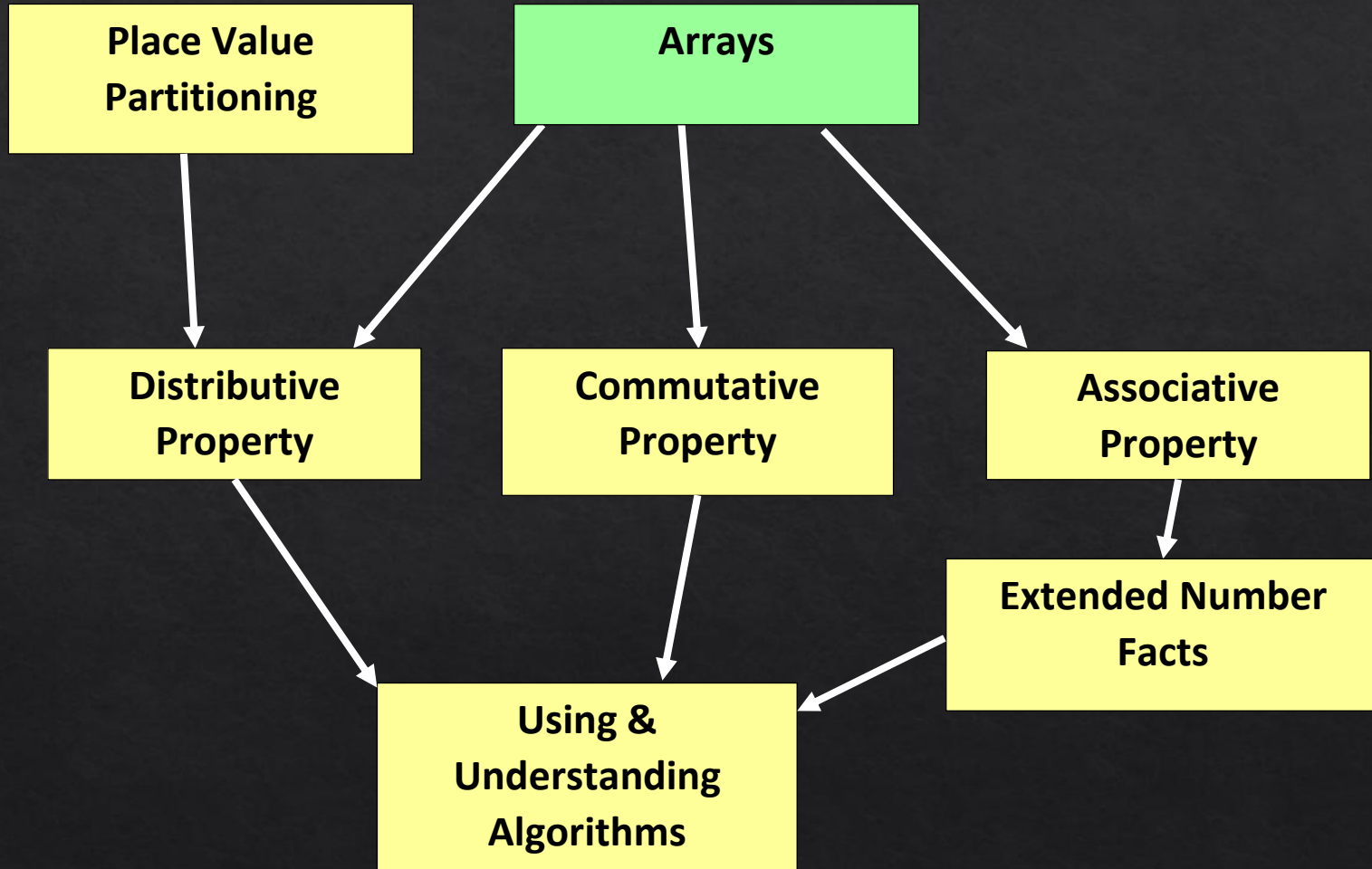
Connections 1



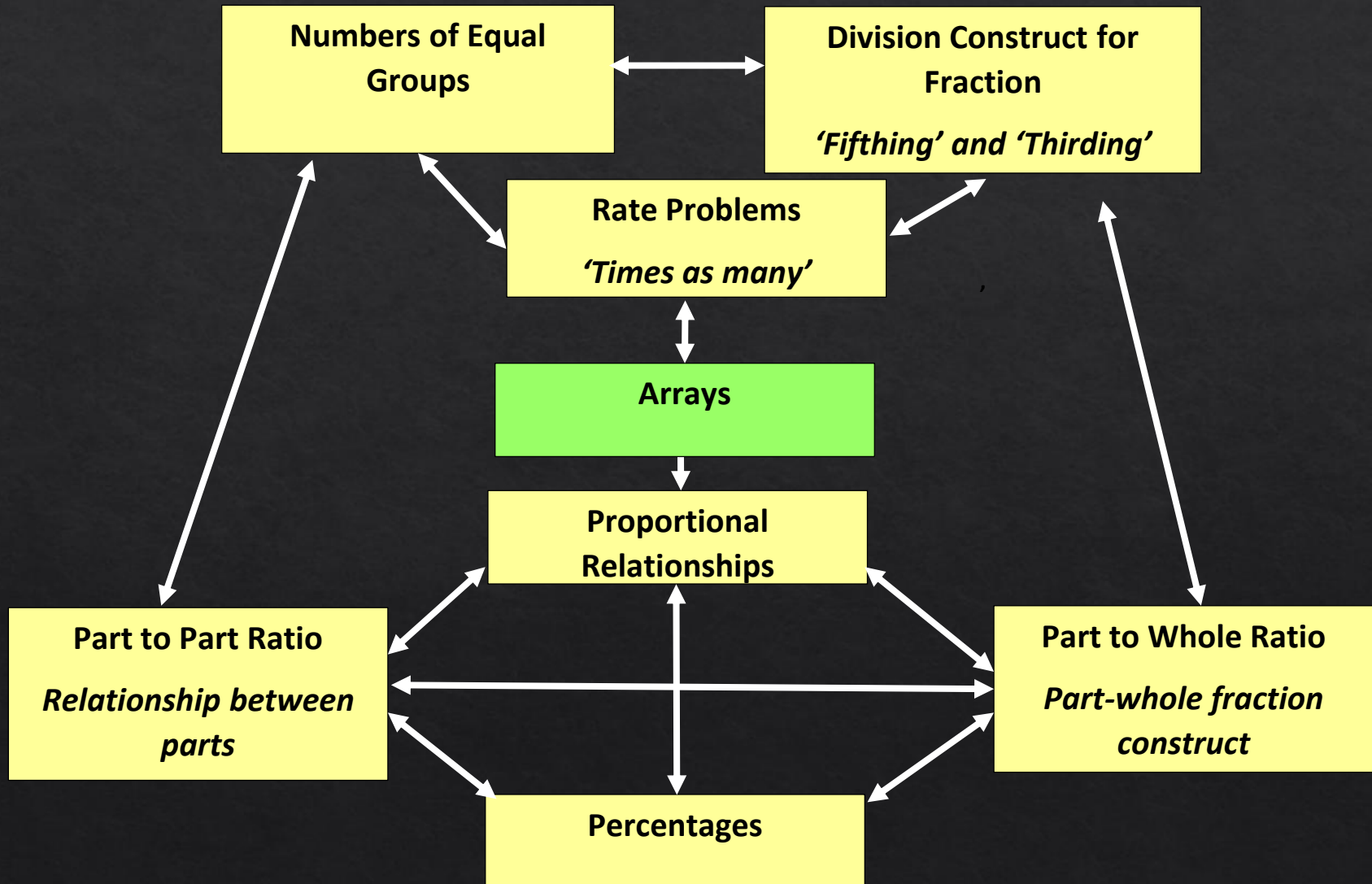
Connections 2



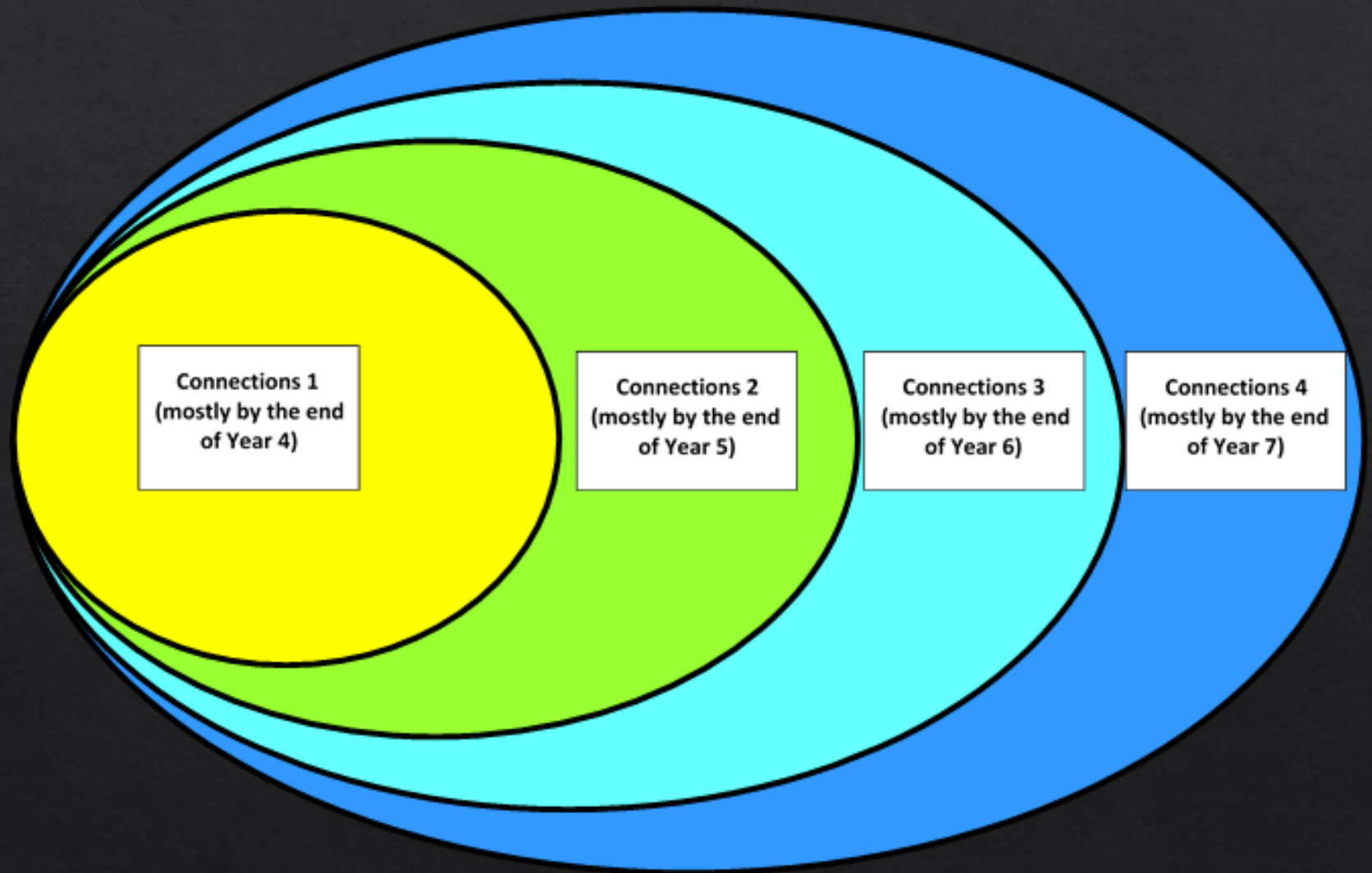
Connections 3



Connections 4

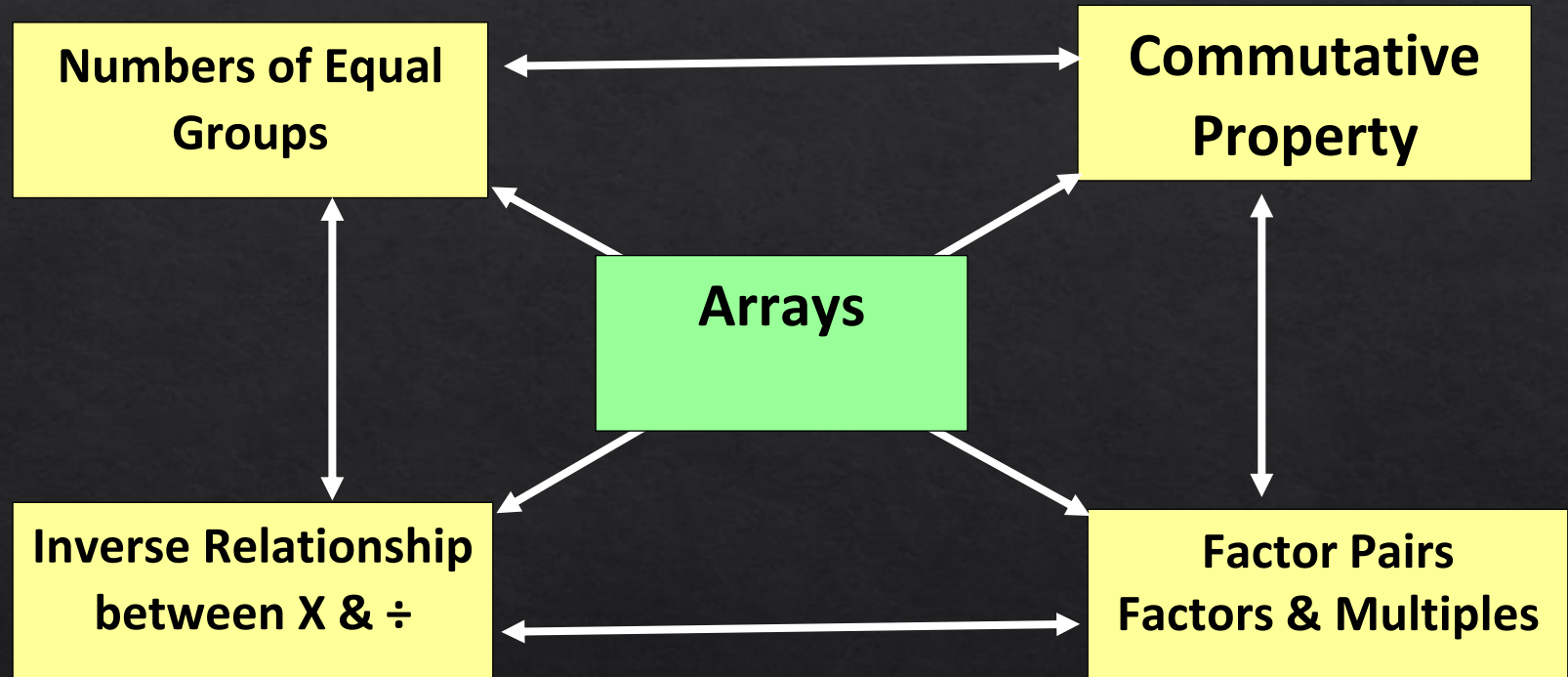


Connections Model

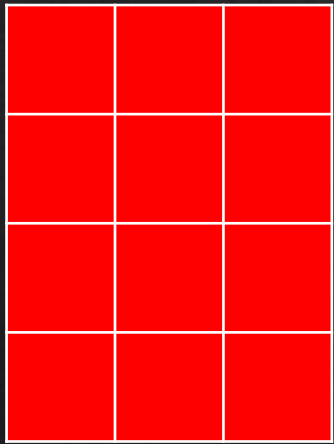


Teachers can be working on any of the connections at any time. That is, some work could be done on the inverse relationship (Connections 1) whilst developing the related notions of digit movement, times as many, and extended number facts (Connections 2), as well as the properties of multiplication (Connections 3) and early fraction understandings (Connections 4).

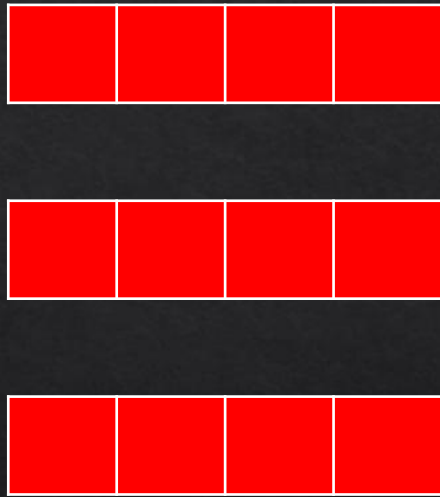
Connections 1



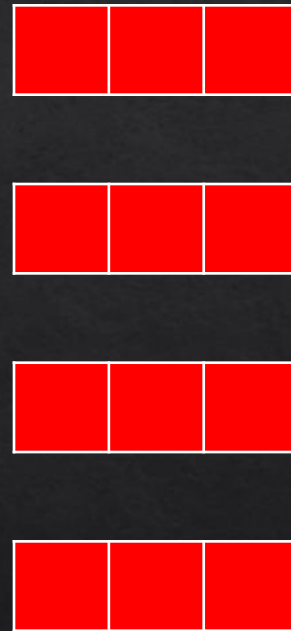
Students were asked to draw a representation of 3×4 . Which answer is the best? Are the answers all correct?



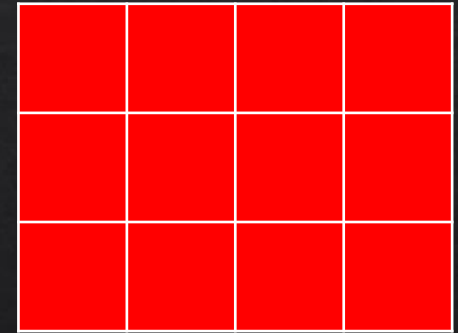
(a)



(b)



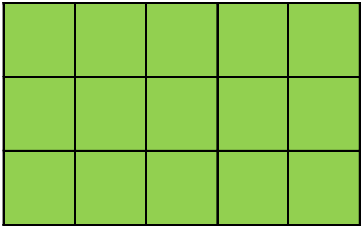
(c)



(d)

Arrays with 24 tiles

- How many different multiplication facts can you make for arrays of 24 tiles?
- What do we call the numbers in the multiplication facts?

<i>Multiplication Fact</i>	<i>Multiplication Story</i>
$3 \times 5 = 15$	John has 3 bags of lollies. In each bag, there are 5 lollies. Altogether, John has 15 lollies.
<i>Array</i>	<i>Division story</i>
	John has 15 lollies. He shared them equally into 3 bags. How many are there in each bag? $15 \div 3 = 5$

Six for the Price of One

$5 \times 3 = 15$ is one number fact but there are other facts related to it.

- What does the number fact mean? (Array etc.)
- What are FIVE other number facts related to this one?

- **Multiplication:** Rotate the 5×3 array to make $3 \times 5 = 15$ (commutative property)
- **Division:** Use the 5×3 array again to make the story that 15 is split into 5 groups with 3 in each group.
- **Fraction:** Make a story that $1/5$ of the total of 15 (each of the 5 rows) means that there are 3 in each row or group. The tile array can be split.
- Rotate the 5×3 array to make a 3×5 array and repeat the above steps.

\$\$\$\$\$\$ Six for the price of one . . . 15

Multiplication	Division	Fraction
$5 \times 3 = 15$	$15 \div 5 = 3$	$1/5 \times 15 = 3$
$3 \times 5 = 15$	$15 \div 3 = 5$	$1/3 \times 15 = 5$

Tell Me a Story

If I had 3 bags each containing 7 lollies, I would have 21 altogether, that is, $3 \times 7 = 21$.

If I had 7 bags each containing 3 lollies, I would have 21 altogether, that is, $7 \times 3 = 21$

Twenty-one apples are put into seven bags. How many are there in each bag? Each bag contains three apples. $21 \div 7 = 3$

Twenty-one apples are put into bags and each bag contains three apples. How many bags are there? There are seven bags.

$$21 \div 3 = 7$$

Tell Me a Story

Use these stories as models and
write six stories.

(2 division, 2 multiplication, 2
fraction)

for this number fact

$$8 \times 4 = 32$$



More Stories

A farmer had 96 sheep and wanted to put them evenly into four pens. How many in each pen?



$$4 \times 24 = 96$$

$$96 \div 24 = 4$$

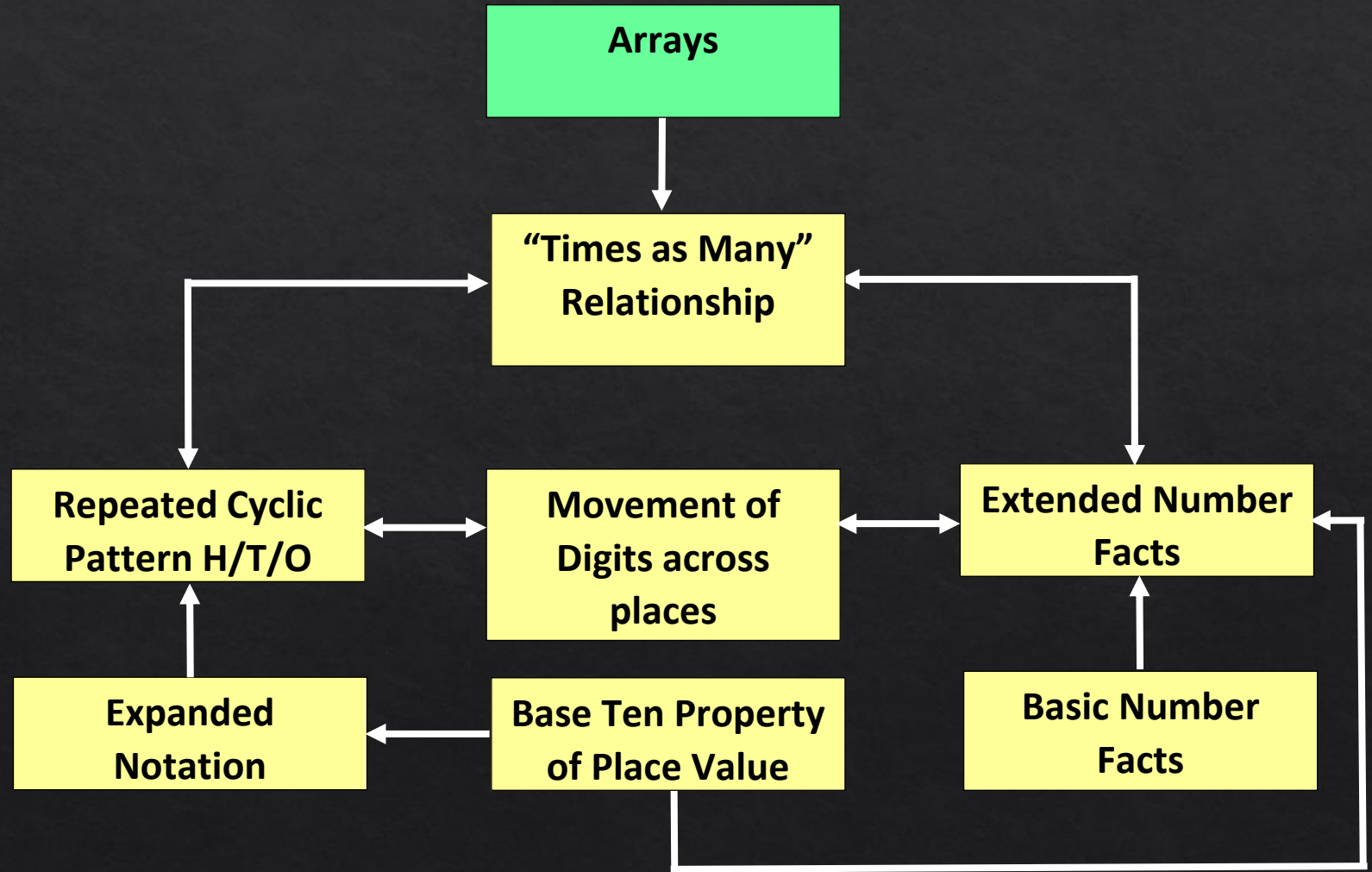
$$96 \times 4 = 384$$

$$\frac{1}{4} \times 24 = 6$$

$$96 \div 4 = 24$$

$$\frac{1}{4} \times 96 = 24$$

Connections 2



Digit movement

- When a number is increased by a power of ten, all of its digits move one place to the left and a zero fills the empty place.
- This needs to be explicitly taught in order to avoid children adopting the 'add a zero' procedure, or when working with decimals, 'moving the decimal point'.

The Marvellous Multiplier

What happened to the 4 when it was multiplied by a power of ten?

Millions			Thousands			Ones		
hundred million	ten million	million	hundred thousand	ten thousand	thousand	hundred	ten	one
100 000 000	10 000 000	1 000 000	100 000	10 000	1000	100	10	1

							4	0
--	--	--	--	--	--	--	---	---

What happened to the 4s when they were divided by a power of ten?
Did the decimal point move?

Researching MT

Many students used 'pseudo-procedures'.

$$3.6 \times 10$$

Why is it not OK to 'add a zero'?

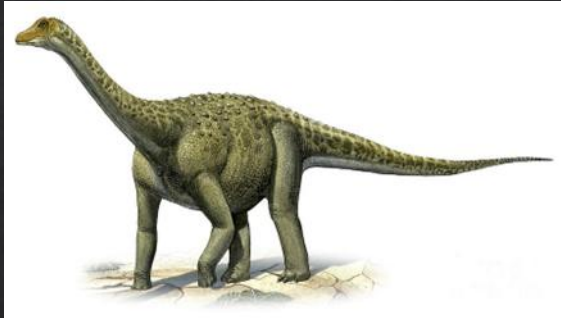
$$3.6 \times 10 = 3.60$$

That's why not!

Measurement

- **Measurement is multiplicative in nature and therefore is a good vehicle for developing multiplicative concepts, in particular, ‘times as many’ and digit movement.**
- **It also provides a great opportunity for some integration across strands in the curriculum.**
- **The place value chart is easily modified to accommodate measurement units.**

Good Task



- A small Hummingbird weighs as little as 3 grams.
- An extinct Titanosaur weighed 150 tonnes.
 - How many times as heavy as the Hummingbird would a Titanosaur have been?

Dinosaur Maths

Most did this . . .

- 150 tonnes = 150 000 kg
- 150 000 kg = 150 000 000 g
- $150\,000\,000 \div 3 =$
50 000 000 times as heavy

This occurred in both the Year 5 and Year 7 classes.

Some useful teaching and learning occurred when students were invited to compare their own weight and height to those of the dinosaurs and other creatures.

150 tonnes



3 grams

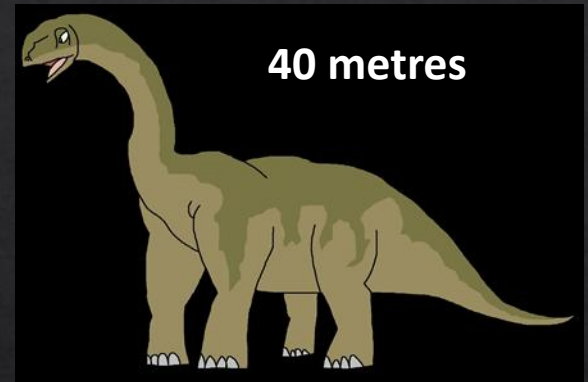


The task also uncovered some place value misunderstandings which led to explicit teaching opportunities.

Dinosaur Maths

- One group of Year 5 boys worked out the 'times as long' relationship by doing $40 \div 0.25 = 160$
- When asked why they did so, their explanation was, "The small dinosaur is 25cm and there are 100 cm in 1 metre, so 25cm. is 0.25m, because the digits have moved back two places".

How many times as long is the large dinosaur compared to the small one?

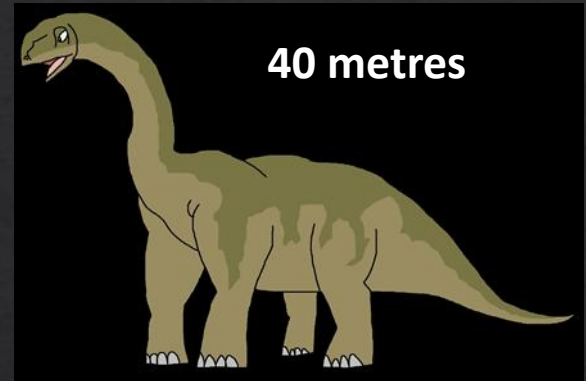


25 centimetres

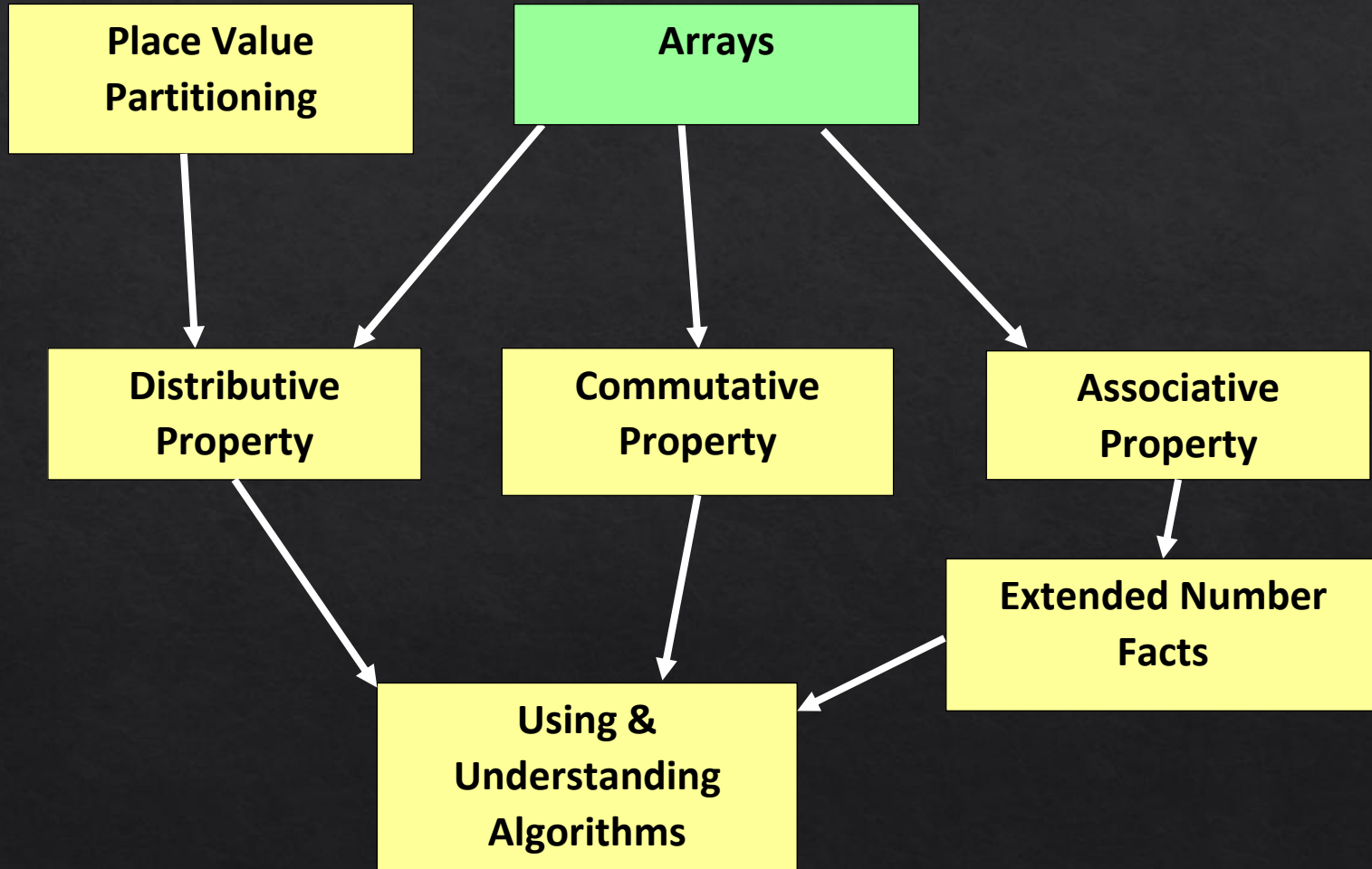
Dinosaur Maths

$$40 \div 0.25 = 160$$

- When quizzed about how they divided and ended up with a larger number than they started with, they said, “We divided by a number less than 1 so the answer must be bigger than 40”.
- They used this method each time they worked out a relationship.



Connections 3



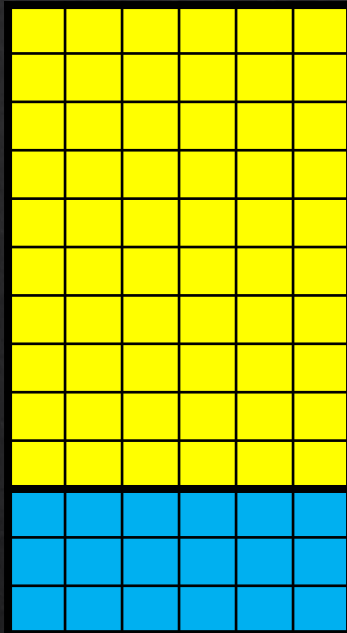
Distributive Property

Mathematics underpinning the Distributive
Property of Multiplication

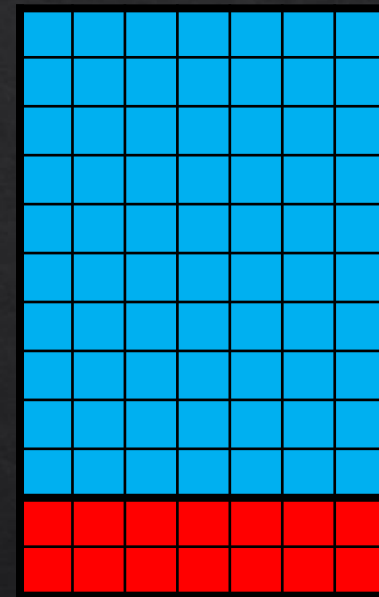
$$27 \times 8 = (20 \times 8) + (7 \times 8)$$

The mathematics underpinning the distributive property provides an excellent example of how multiplicative thinking is an issue that begins well before, and extends well beyond the middle primary years.

Multiplication and Division Tiles



Write two number sentences
to describe your array.

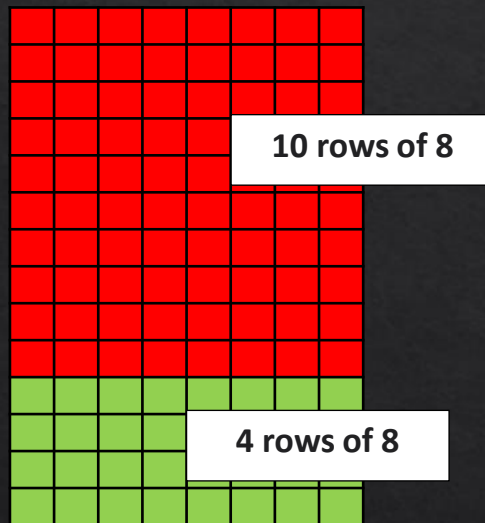


$$84 \div 7$$

$$(10 \times 7 = 70) + (2 \times 7 = 14)$$

$$(10 \times 7) + (2 \times 7) = (10 + 2) \times 7 = 12 \times 7 = 84$$

Multiplication and Division Tiles

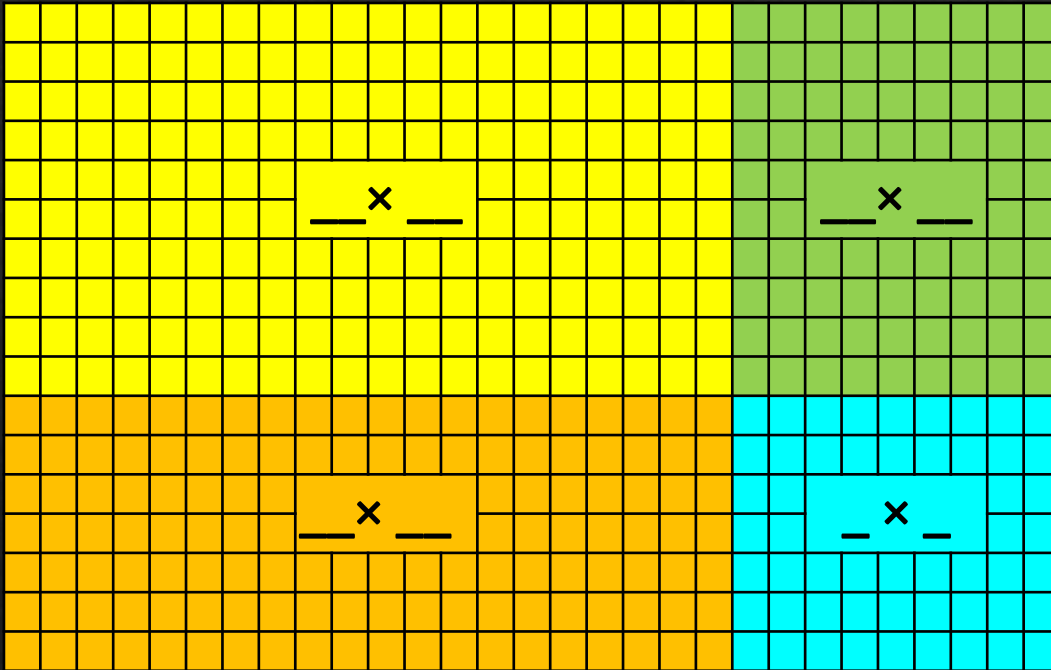


$$14 \times 8$$

14 x 8	
10 x 8	4 x 8
80	32
112	

Arrays to Algorithms

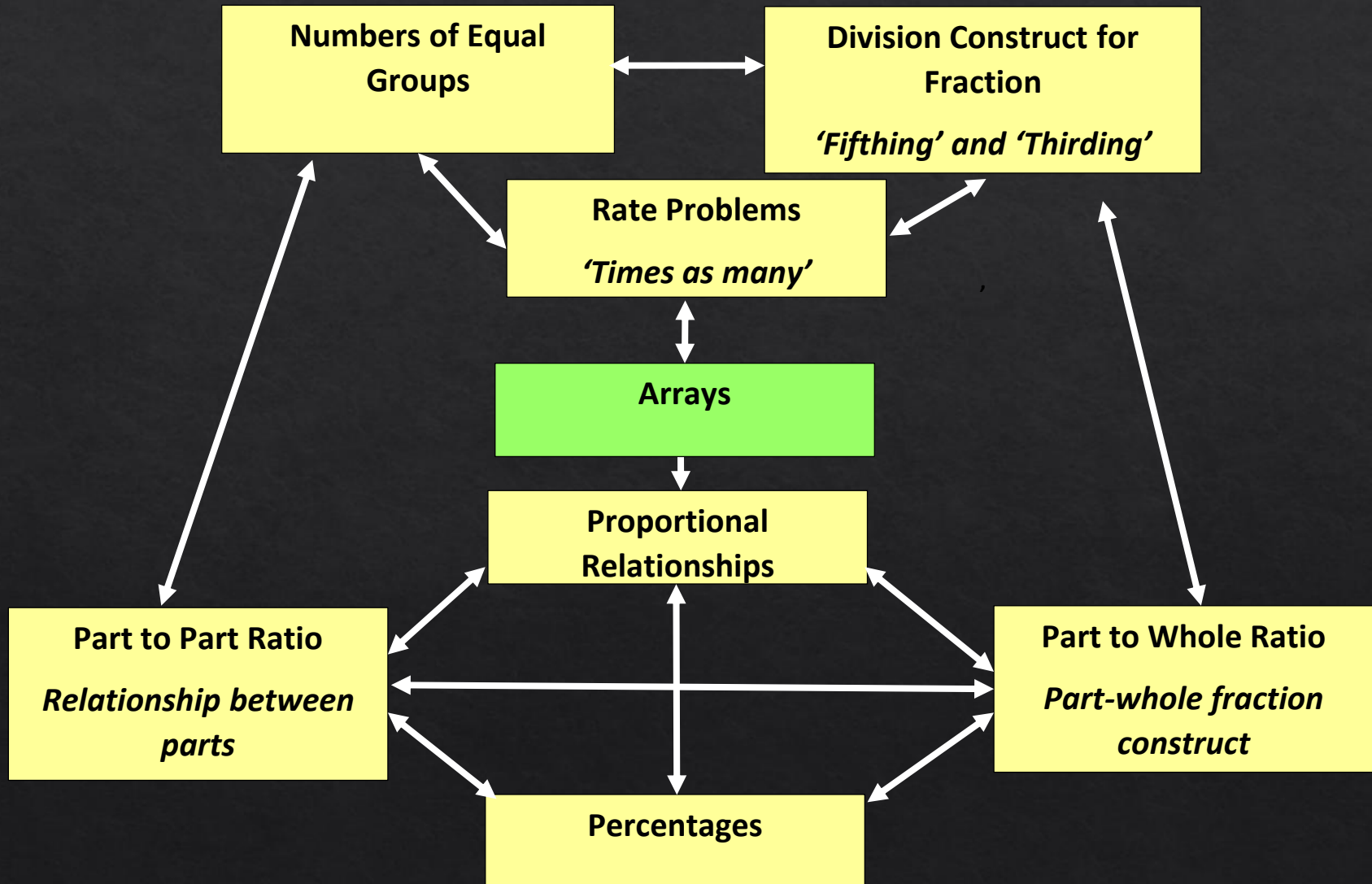
17 X 29



	1	7	
x	2	9	
			x
			x
			x
			x

6			
	1	7	
x	2	9	
			x
			x

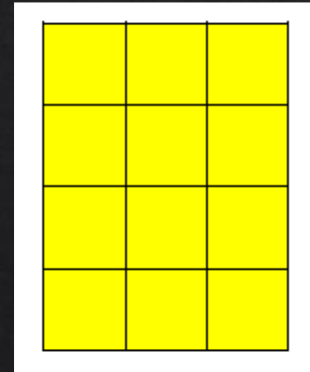
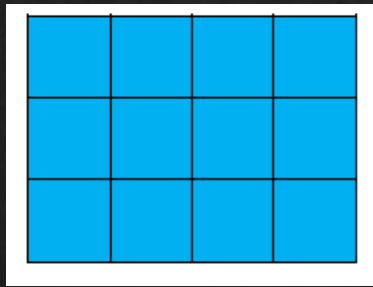
Connections 4



Equivatiles

Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$?

Make a 3×4 array in one colour
and a 4×3 array in another colour.

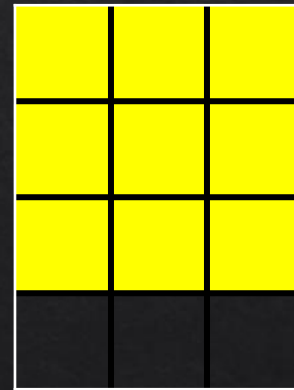


Equivatiles

Identify the tiles that make up $\frac{2}{3}$ of the first array and $\frac{3}{4}$ of the second array, by removing one row of each array ($\frac{1}{3}$ in the first array and $\frac{1}{4}$ in the second array).



$$\frac{2}{3}$$

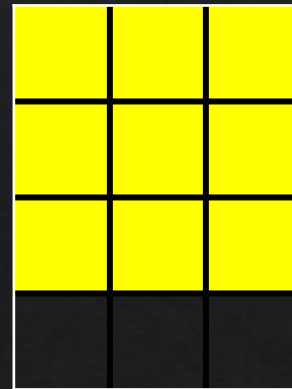


$$\frac{3}{4}$$

Equivatiles

- Ask the question, “Which has the most tiles, $\frac{2}{3}$ or $\frac{3}{4}$?”
- Discuss other names for the two fractions arriving at $\frac{8}{12}$ and $\frac{9}{12}$.
- Discuss that they are equivalent fractions which can be derived by finding the lowest common denominator.

$\frac{2}{3}$

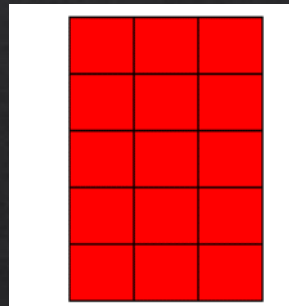


$\frac{3}{4}$

Amazing Tiles



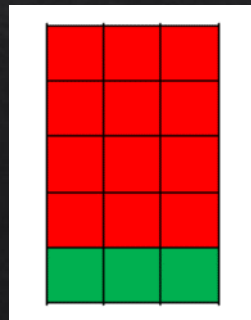
Make a 5×3 array in one colour.



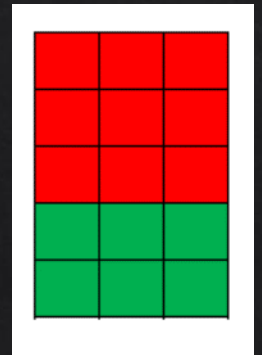
Ask, "What fraction is represented by the red tiles?"

Arrive at $15/15$, $5/5$ (5 rows), $3/3$ (if the array is rotated).

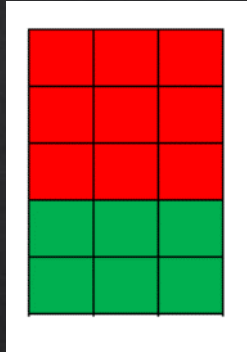
Replace one row with tiles of another colour. Discuss the various fractions shown in the array



Repeat the process and replace another row.



Amazing Tiles



- Ask them to record what they see in these ways:

- $3/5 + 2/5 = \underline{\hspace{2cm}}$

- $3/5 + \underline{\hspace{2cm}} = 5/5$

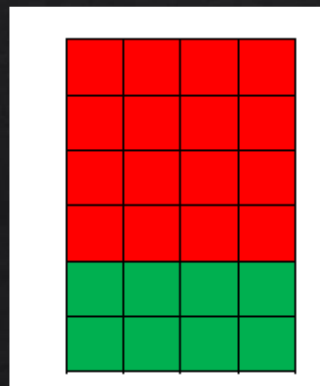
- $\underline{\hspace{2cm}} + 2/5 = 5/5$

- $9/15 + 6/15 = \underline{\hspace{2cm}}$

- $9/15 + \underline{\hspace{2cm}} = 15/15$

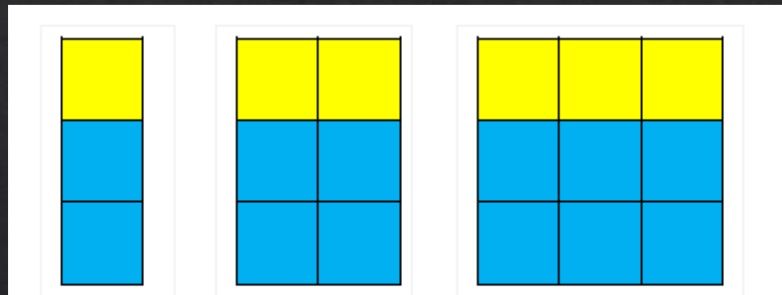
- $\underline{\hspace{2cm}} + 6/15 = 15/15$

Discuss everything
'fractional' about
this array.



Amazing Tiles

Make these arrays using tiles of two different colours.



Does each array show one third?

How do you know?

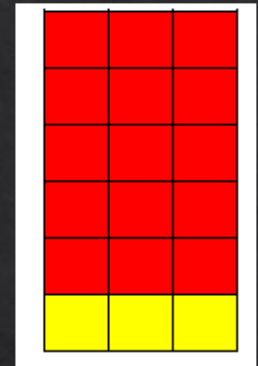
What else does each array also show?

Amazing Tiles

Make a 6×3 array using tiles of two colours.

What is the part-to-part ratio for the two tile colours.

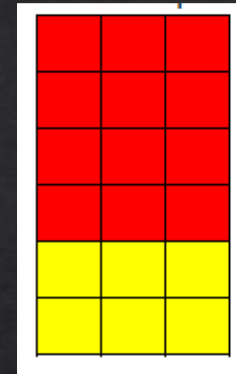
Establish that, for red to yellow, it can be $15 : 3$, or $5 : 1$



Also, note the ratio for yellow to red is the reverse, that is $3 : 15$ or $1 : 5$

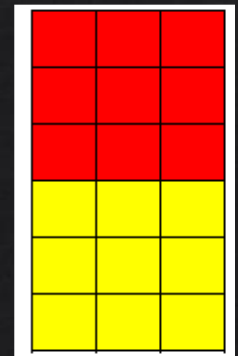
Amazing Tiles

Replace another row and derive the part-to-part ratios for red to yellow – 12 : 6 or 4 : 2. Some may also see the ratio of 2 : 1.

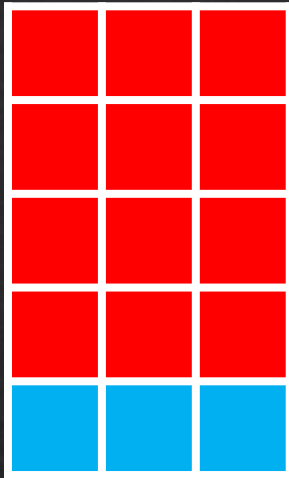


Also, note the ratio for yellow to red is the reverse, that is 6 : 12 or 2 : 4. Some may also see the ratio of 1 : 2.

Replace another row of tiles and repeat the process. Note the ratio of 1 : 1.



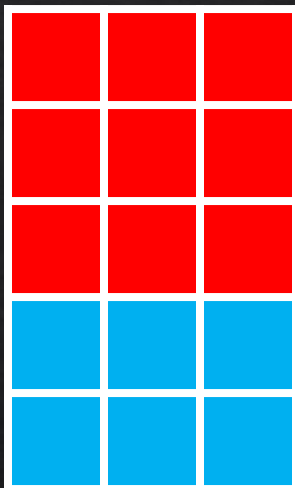
Amazing Tiles



Make a 5×3 array using two colours.

Ask what the part-to-whole ratio is for each colour.

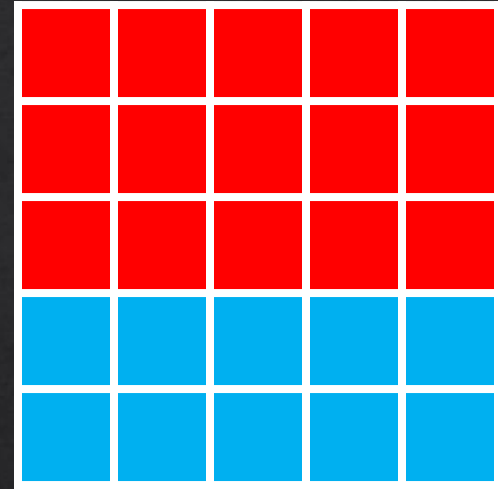
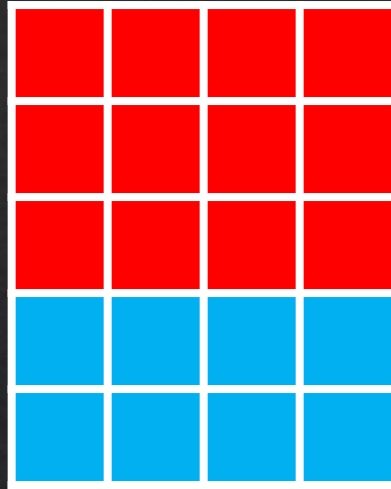
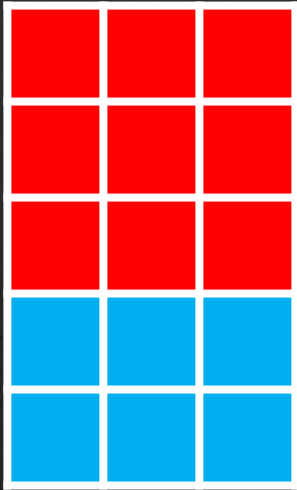
For red, it could be $12 : 15$ or $4 : 5$, and for blue, $3 : 15$ or $1 : 5$.



Replace another row with tiles of the lesser colour

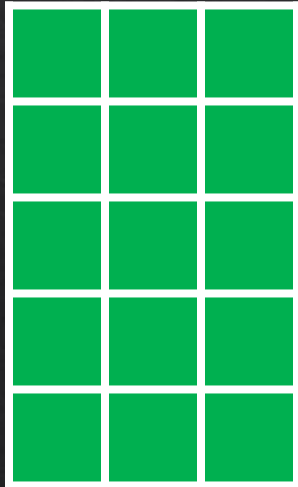
Establish the part-to-whole ratio for each colour.

Amazing Tiles



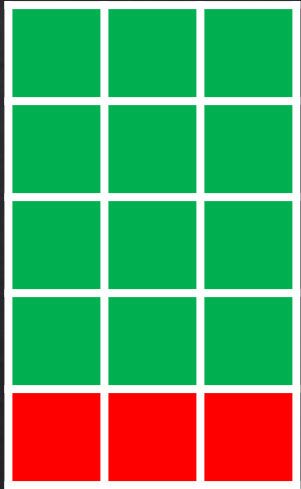
- In stages, increase the length of the rows by one tile.
- Derive the part-to-whole ratio for each colour in as many ways as possible.
- Ask, “What changes in the part-to-whole ratio, and what stays the same?”

Amazing Tiles



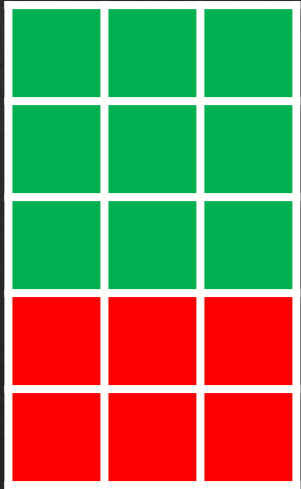
- Make a 5×3 array using tiles of one colour.
- Ask, “What percentage of the tiles are green?”
- Derive that it is 100%.
- Pose the question, “But how can that be if there are not 100 tiles?” Discuss.

Amazing Tiles



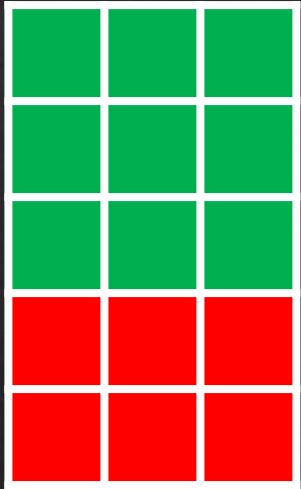
- Replace one row of tiles with tiles of another colour.
- Discuss the percentage of the array in terms of the tile colours.
- Derive that each row is 20% of the array.
- Develop the notion of 'gap thinking' or residual thinking'. If 20% of the tiles are red, then 80% must be green.

Amazing Tiles

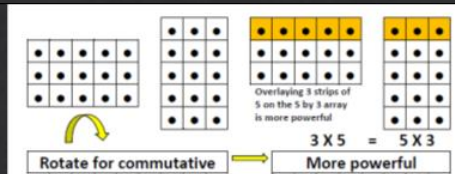
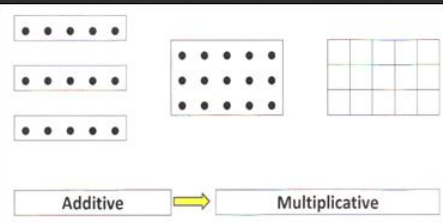


- Replace another row of tiles with tiles of the other colour.
- Discuss the percentage of the array in terms of the tile colours.
- Develop the notion of 'gap thinking' or residual thinking'. If 40% of the tiles are red, then 60% must be green.

Amazing Tiles



- Make links to part-to-part ratio, part to whole ratio, and equivalent fractions using this array.
- For example, the part-to-part ratio for green to red is 9 : 6, or 3 : 2.
- The part-to-whole ratio for green is 9 : 15 or 3 : 5.
- The fraction of green tiles is $\frac{9}{15}$ or $\frac{3}{5}$ and for red, it is $\frac{6}{15}$ or $\frac{2}{5}$



Factor of 15	Factor of 15	Multiple of 3 & 5	
Number of groups	Number in each group	Total of all groups	
5	X	3	= 15
5 is scaled up by a factor of 3			

Multiple of 3 & 5	Factor of 15	Factor of 15
Total of all groups	Number of groups	Number in each group
15	÷	5 = 3
15 is scaled down by a factor of 5		

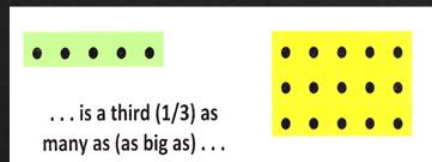
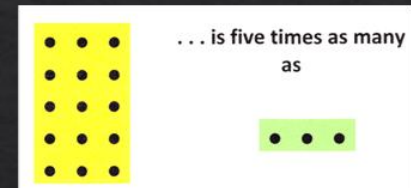
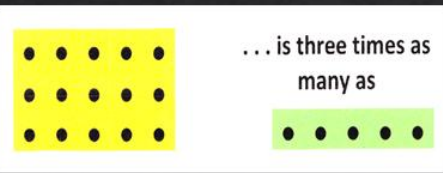
1/5	● ● ●	3/15
1/5	● ● ●	3/15
1/5	● ● ●	3/15
1/5	● ● ●	3/15
1/5	● ● ●	3/15

1/5 of 15 = 3	● ● ●	The notion of "fifthing"
1/5 of 15 = 3	● ● ●	
1/5 of 15 = 3	● ● ●	
1/5 of 15 = 3	● ● ●	
1/5 of 15 = 3	● ● ●	

1/3	● ● ● ● ●	5/15
1/3	● ● ● ● ●	5/15
1/3	● ● ● ● ●	5/15

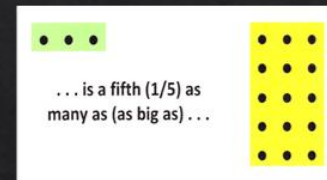
1/3 of 15 = 5	● ● ● ● ●	The notion of "thirding"
1/3 of 15 = 5	● ● ● ● ●	
1/3 of 15 = 5	● ● ● ● ●	

The Multiplicative Array
Making Explicit Connections
 © Chris Hurst, 2015



$\frac{15}{3} = \frac{5}{1}$	$\frac{15}{5} = \frac{3}{1}$
$\frac{3}{15} = \frac{1}{5}$	$\frac{5}{15} = \frac{1}{3}$

Area of this 3 x 5 array is 15 square units



Distributive Property	
(10 X 6) + (3 X 6) = 78	
10 X 6	3 X 6

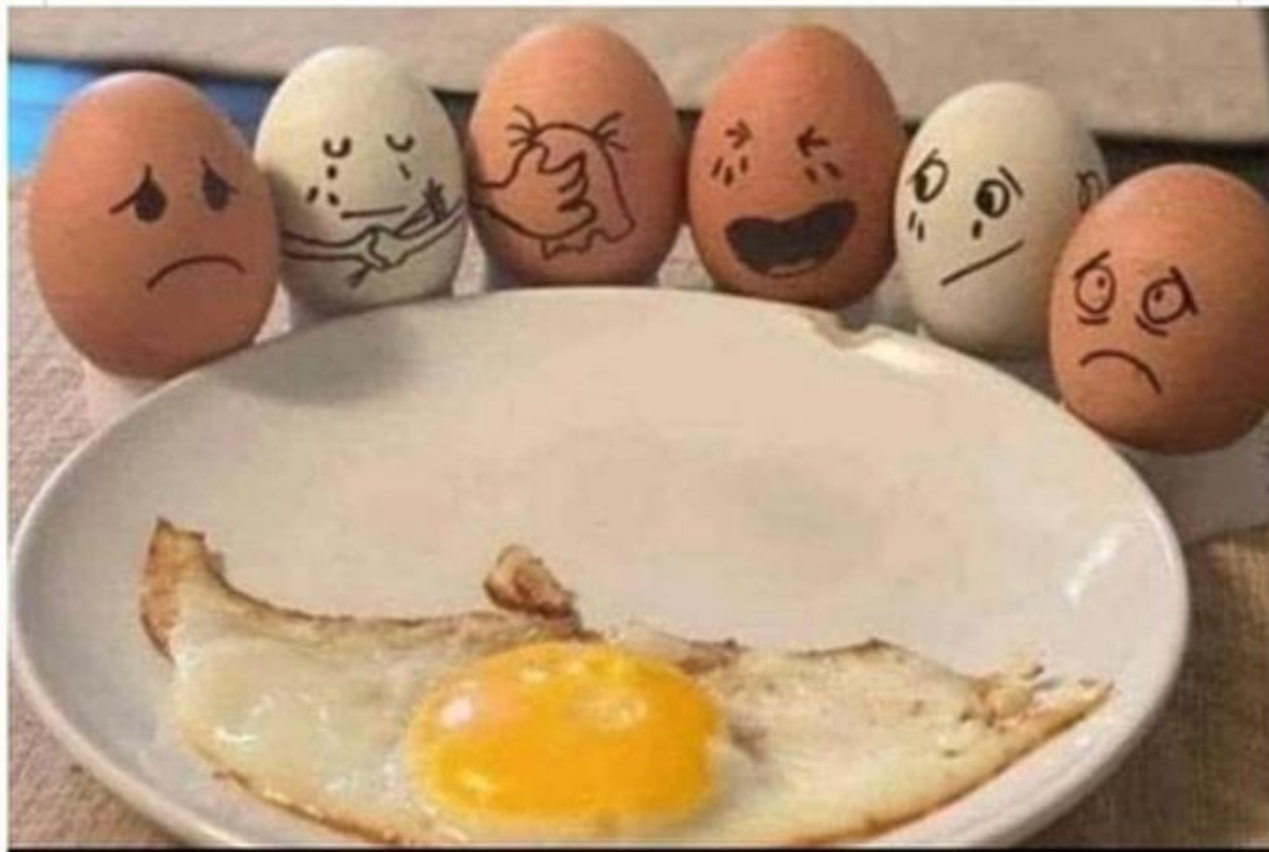
Ratio of part:part is 1:2
 That is 1/3 and 2/3 or 5/15 and 10/15

Ratio of part:whole is 1:3 or 5:15

How many different combinations can you have if there are five choices of fruit (P, S, B, C and O) and three choices of drink (C, T and F)?

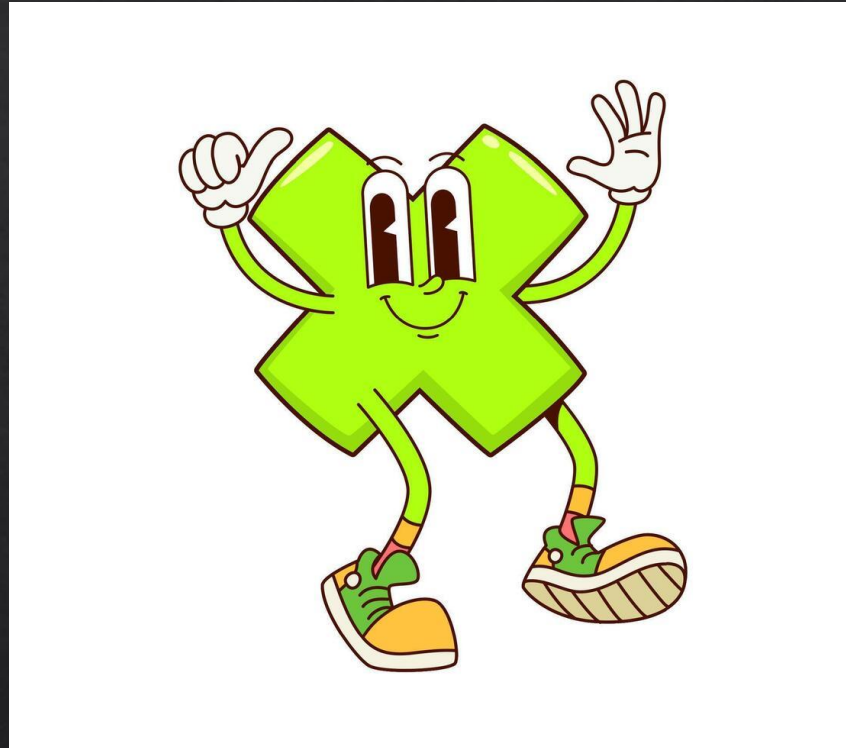
	Food				
	P/C	S/C	B/C	C/C	O/C
	P/T	S/T	B/T	C/T	O/T
	P/F	S/F	B/F	C/F	O/F

3 x 5 = 15



He died last fry day. Thank God he wasn't beaten. Don't worry, he went over easy. He's now on the sunny side. He's definitely in a better plate.

Think Multiplicatively!



Thank You!